Panchanga-Tantra

The Magic of the Indian Calendar System

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Foreword to the Second Edition

The fable of Apara Ganita and the Mystical Garden of Enchanted Numbers is obviously fictional. The inspiration is *Leelavati Ganitam,* a chapter in the ancient mathematical treatise, the *Siddhanta Siromani,* written by Bhaskaracharya in 1150CE. The *Leelavati Ganitam* is fascinating not only for its treatment of indeterminate analysis and a method to solve Pell's Equation, but also, as a Canadian university's website on mathematical history puts it, for its poetic conversation between the narrator and a narratee named Leelavati[1]. The similarity between this poetic construct and the conversation between Apara Ganita and the *dwara palika* is probably noticeable.

Frame stories are not common for scientific research papers, but they certainly have a historical precedent.

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[1] "Bhaskaracharya", *History of Mathematics,* Simon Fraser University, <http://www.math.sfu.ca/histmath/India/12thCenturyAD/Bhaskara.html> (21st September, 2002.)

Prologue – The Mystical Garden of Enchanted Numbers

Once upon a time, in the magical mystical city of Suvarnapuri[2]*, there lived a student called Apara Ganita*[3]*. Apara Ganita was virtuous and devoted to his sciences. Having spent considerable amount of time learning the shastras from his guru, he was surprised when one day his guru called him up.*

"You have performed well, O sishya[4] *mine", the guru said, "but the time has now come for you to take leave".*

Apara Ganita was at once sad, for he had learned a lot under him. But he remained quiet and continued listening to his guru.

"Listen, Apara Ganita, I shall now tell you something that my guru told me when I finished my studies. For, a study in Ganita Sastra (mathematics) is not complete, unless one visits the Mystical Garden of Enchanted Numbers"

"You must go and find this place for your education to be truly complete".

And so Apara Ganita went about searching for this place. Indeed, after much travelling and searching, he was finally shown the way to the Mystical Garden of Enchanted Numbers.

And lo, what a beautiful sight it was! For it was situated in the midst of a lush green valley, saddled by mountains on either side. Down there, Apara Ganita could see famous mathematicians expositing their theories and skills, like hawkers on a bazaar street. There was Euclid standing on a rectangle, explaining the beauty of the Golden Ratio in classic Greco Caldean architecture. Pythagoras was standing next to him as a

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[2] Suvrnapuri = City of Gold
[3] Apara Ganita = someone with a lot of mathematical talent.
[4] Sishya = student

part of the Greek exhibit, explaining the virtues of a right-angled triangle to a curious crowd. From the far end of the Orator's Corner, Zhao Jun Qing looked at Pythagoras and smiled. He was himself holding a right-angled triangle and was explaining his proof for the Pythagoras' Theorem. Mandelbrot was decorating the Garden with flowers of fractallate beauty. John Nash was close by; he was pointing at a group of women, probably explaining game theory to onlookers around him. In another corner of the garden, (Sector 1729), Srinivasa Ramanujan was vociferously arguing a point with Thomas Hardy.

It was such an environment that Apara Ganita wanted to enter.

However, as he was about to enter through the great doors guarding the garden, he heard a sonorous voice calling out his name.

He stopped and turned around to see who was calling him only to saw a young woman coming towards him. With eyes burning with curiosity and a voice sweeter than a nightingale, she said: -

O Student Erudite,
What is it that you study tonight?

Just what I needed, a mystical dwara palika *(female door keeper), he said to himself. Shaking his head in wry amusement, he looks at the books in his hand and takes a deep breath to begin his dissertation....*

Contents

Sthaana Prakarana – How the calendar is different in different regions

In a sonorous voice, the dwara palika said,
"In order to ascertain your dissertation's veracity,
can I hear you talk about the calendar's regional complexity?"
To which Apara Ganita listened to the multitudes of voices in the Garden, and replied thus:-

Probably the easiest way to classify Indian calendars is by the region of usage. It must be reiterated though, that such an exercise might be misleading. The classification is indeed not watertight; all calendars are intrinsically inter-linked with one another. A flowchart of the various Indian calendars and the links between them is given in the Appendix.

With this caveat, we'll now traverse India on a calendrical vehicle of sorts. In particular, we try to ascertain the following elements in each region's calendrical practices: -

- Basis of the Calendar
- Local Variation.
- When does the year begin?
- Era Followed

We'll find the following calendars defined with these metrics: -

The Southern Amaanta Calendar

The Southern Amaanta Lunisolar Calendar is predominantly followed in the South and South-West Indian states of Andhra Pradesh, Karnataka and Maharashtra. It is essentially a lunisolar one; i.e., its days and months are calculated based on the motions of the moon. Like the Chinese calendar, the month is calculated from new moon to new moon. It however, differs from the Chinese calendar in that the lunar day ("*thithi*") of the new moon is considered the last day of the previous month. Again, as in

the Chinese calendar, a leap month, an *adhika maasa,* is added to the calendar every 2.7 years on an average to offset the disparity in lengths between the lunar year and the sidereal year. In addition, a month, the *kshaya maasa*, is occasionally subtracted. This is discussed in a later segment.

The Southern Amaanta Calendar differs from the Western Amaanta Calendar in its treatment of *kshaya maasas*, the New Year Day and the Era followed. We believe that the Southern Amaanta Calendar follows the Southern School for treating kshaya maasas. Saha and Lahiri suggest that it follows the Salivahana Saka Era starting with Chaitra Sukla Pratipada[5], the lunar day after the last new moon before *Mesha Sankranti*. The years are also named according to the names of the Jovian years (Southern School[6]). The Eras and handling of kshaya maasas will be discussed in detail in their respective segments.

Western Amaanta Calendar

As already mentioned, we believe it's important to distinguish between the Amaanta calendar practised in South and West India. In West India, specifically, in the state of Gujarat, the Amaanta calendar is of two forms[7], one that starts with Aashaadha (followed in the Kathiawar region) and one that starts with Kartika (followed all throughout Gujarat). Both calendars follow the Vikrama Era and both also possibly follow the North Western School for *kshaya* months.

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[5] Chakravarty, Apurba Kumar and SK Chatterjee "Indian Calendar from Post-Vedic Period to AD 1900" in History of Astronomy in India. (1985: New Delhi) Indian National Science Academy. p. 304

[6] Saha and Lahiri. Report of the Calendar Reform Committee. (1985: New Delhi) Indian National Science Academy. p. 270

[7] Chakravarty et al. p. 304

Purnimaanta Calendar

The Purnimaanta Calendar is followed in most of North India, i.e., in the states of Bihar, Himachal Pradesh, Uttar Pradesh, Haryana, Punjab, Jammu and Kashmir and Rajasthan[8]. (Earlier literature fails to mention Uttaranchal, Chattisgarh, Jharkhand and Delhi, but they are off-shots of bigger states and would necessarily follow the same calendar). The last of the three Indian lunisolar calendars, this one differs from the Amaanta calendar in that the months are reckoned from full moon to full moon. Therefore, the Purnimaanta calendar starts two weeks before the Amaanta calendar does; that is, it starts with the lunar day after the last full-moon before Mesha Sankranti. The Vikrama Era is followed[9], along with the Northern School of Jovian-year names[10].

The Malayali Calendar

We now come to the list of Solar Calendars. The Malayali Calendar is predominantly followed in the South Indian state of Kerala. It is essentially a solar calendar; as we shall see later, the months are defined according to the *raasis*. The year starts with the Simha Sankranti and follows the Kollam Era.[11] The month begins on the same day as a Sankranti if it occurs before *aparahna,* i.e., three-fifths of a day. Otherwise, it begins on the next day.

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[8] Chatterjee, SK. Indian Calendars. p. 42
[9] Chakravarty et al. p. 305
[10] Saha et al. p. 270
[11] Chakravarty et al. p. 304

Tamil Calendar

The Tamil calendar is followed in Tamil Nadu. This calendar is also solar; the month begins on the same day as a Sankranti if it occurs before sunset[12]. The Kali Era is followed along with the Southern Jovian cycle. One peculiarity about the Tamil calendar is that its month names start with Chittirai[13] (Chaitra).

Bengali Calendar

The Bengali calendar is predominantly followed in West Bengal, Assam and Tripura. The Era is the Bengali San. The rule for the beginning of the month is again different; the month begins on the day after a Sankranti, if it occurs before midnight. Otherwise, it begins on the third day.[14]

Oriya Calendar

The Oriya calendar is followed in the Eastern state of Orissa. In addition to the Bengali San, the Saka, Vilayati and Amli eras are followed.[15] The month begins on the same day as that of the respective Sankranti.[16]

The Nanakshahi Calendar

Promulgated in 1998 CE, the Nanakshahi Calendar is followed in Punjab. It's intrinsically linked to the Gregorian calendar, except in its usage of the Nanakshahi Era.[17]

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[12] Chatterjee. p. 14
[13] Ibid. p. 9
[14] Ibid. p. 14
[15] Saha et al. p. 258
[16] Chatterjee. p.14
[17] Pal Singh Purewal, Nanakshahi Samat < http://www.sikh.net/sikhism/Nanakshahi.htm > (22nd September, 2002)

National Calendar of 1957

Proposed by the Calendar Reform Committee of 1952 and promulgated in 1957 CE, the National Calendar is a tropical calendar with fixed lengths of days and months. However, because it was totally different from the traditional calendars, it did not find much acceptance.[18]

We may thus summarize Indian calendars thus: -

State	Calendar	Era	New Year	Further Local Variation
Andhra Pradesh	Southern Amaanta	Salivahana Saka, Jovian cycle (Southern School)	One day after the last new moon before Mesha Sankranti	Possible *Jugma* month for kshaya
Assam	Solar	Kali, Bengali San	Solar Day after Mesha Sankranti	Bengali rules for beginning of month
Bihar	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
Chattisgarh	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
Delhi	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
Goa	Southern Amaanta	Salivahana Saka, Jovian cycle (Southern School)	One day after the last new moon before Mesha Sankranti	Possible Jugma month for kshaya
Gujarat	Western Amaanta	Vikrama Karthikaadi	One day after Deepavali	North-western school for kshaya possible

[18] Chatterjee. p. 19

Himachal Pradesh	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
Jammu and Kashmir	Purnimaanta	Saptarishi, Laukika	One day after the last full moon before Mesha Sankranti	
Jharkhand	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
(Kathiawar)	Western Amaanta	Vikrama Aashaadhadi	Ashaadha S 1	North-western school for kshaya possible
Karnataka	Southern Amaanta	Salivahana Saka, Jovian cycle (Southern School)	One day after the last new moon before Mesha Sankranti	Possible Jugma month for kshaya
Kerala	Solar	Kollam Era	Simha Sankranti	1) Kerala rules for beginning of month 2) Months named after raasis
Madhya Pradesh	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
Maharashtra	Southern Amaanta	Salivahana Saka, Jovian cycle (Southern School)	One day after the last new moon before Mesha Sankranti	Possible Jugma month for kshaya
Orissa	Solar	Saka, Vilaayati, Aamli, Bengali San	Mesha Sankranti	
Punjab	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
Punjab – Nanakshahi	Sidereal; fixed relative to Gregorian calendar	Nanakshahi	14th March	Uses the traditional names for Punjabi months
Rajasthan	Purnimaanta	Vikrama Era	One day after the	

		(Chaitradi)	last full moon before Mesha Sankranti	
Tamil Nadu	Solar	Kali, Jovian cycle (Southern School)	Mesha Sankranti	
Tripura	Solar	Kali, Bengali San	Solar Day after Mesha Sankranti	Bengali rules for beginning of month
Uttaranchal	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
Uttar Pradesh	Purnimaanta	Vikrama Era (Chaitradi)	One day after the last full moon before Mesha Sankranti	
West Bengal	Solar	Kali, Bengali San	Solar Day after Mesha Sankranti	Bengali rules for beginning of month

Table 1: - Calendrical practices in different Indian states

Note:

1) The table is exhaustive neither in terms of calendars nor in terms of states. Arunachal Pradesh, Manipur, Meghalaya, Mizoram, Nagaland and Sikkim were left out.

2) Chatterjee mentions that the Orissa School for deciding the beginning of the solar month is also used in Punjab and Haryana "where the solar calendar is also used".[19]

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[19] Chatterjee, SK p. 14

Maasa Naamakarana - How the Months got their Names.

Listening to this, she said,

"Since we are deep in this game,
Might I ask how each month got its name?"

To which Apara Ganita stared at a gulmohar flower with twenty-seven buds and replied thus:-

The complexity of the Indian calendar system is not just in the plethora of calendars available, but also in the manner in which they link up with one another. A principal point of linkage of most Indian calendars is in their names of the months; as we shall see, the similar sets of month names are used in more than one calendar. In this section, we aim to formulate rules determining the naming of the months. Our motivation is not just taxonomic; month names, we shall see, are critical to understanding the Indian calendar system.

We propose that there are two types of month names: -

1) Months named after *Nakshatras*

The set of month names named after *nakshatras* is used by both solar and lunisolar calendars, adding to seeming complexity of the Indian calendar system. Indeed, as we shall see, this type should actually called as 'Months *initially* named after *Nakshatras'*; there has been an infusion of solar rules into an essentially lunar convention.

Let us then, first consider the original rule. Saha and Lahiri mention that *pakshas* or fortnights were differentiated based on the *nakshatra "*where the moon is full".[20] That is to say, if a particular full moon occurs near, say, the lunar asterism, *Visakha*, the full moon would be called as *Vaisakha Purnimaasi*, and the month would be *Vaisakha.* The earliest lunisolar months, then, were *purnimaanta*, that is, the name of the full moon

[20] Saha et al. p. 221

corresponded to the name of the month. Of course, the full moon occurs at all *nakshatras*. Fifteen were taken into account for naming of the month, spaced more or less equally.

We thus have the following set of names along with their respective *nakshatras*[21]*:*

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Nakshatra on Purnima	Month Name
Chitra	Chaitra
Visakha	Vaisakha
Jyestha	Jyaistha
(Purva & Uttara) Aashaadha	Aashaadha
Sravana	Sraavana
(Uttara & Purva) Bhaadrapada	Bhaadrapada
Asvini	Asvayuja (Aasvina)
Krittika	Kaarthika
Mrugasira	Maarghasira
Pushyami	Pausa (Pushyam)
Maghaa	Maagha
(Uttara and Purva) Phalguni	Phalguna

It may be noted that the months of *Aashaadha, Bhadrapada* and *Phalguna* are linked to two *nakshatras* respectively. Chatterjee and Chakravarthy give the following criteria for choosing *nakshatras* for month names[22]: -

1) The *yogataaras* or the identifying stars of the *nakshatras* are prominent or have traditional significance.
2) They are spaced more or less equidistant from one another.

It must be mentioned that this rule is now an *approximation* largely due to Earth's precession; for instance, this year's *Chitra Purnimaasi* had *Swati* as its *nakshatra*. Also, possibly for historical reasons, and allowing for regional variation in pronunciation, the Oriya, Bengali, Assamese, Punjabi and Tamil solar calendars also use the same set of month names. To reconcile all this, we might frame a new rule; that, the *amaanta* lunar

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[21]Saha et al. p. 221
[22] Chakravarthy et al, p. 281

month takes its number from the solar month that starts in it, but its name from the solar month in which it starts, while following the *purnimaanta* months in chronological order. That is to say, since *Chitra* occurred during the *purnima* of this year's first *purnimaanta* month, we call this month as 'Chaitra'. Consequently, the first *amaanta* month would also be 'Chaitra', which also would be the name of the solar month during which the a*maanta* 'Chaitra' started. However, the 'number' of the solar month ('1' in the case of *amaanta* and *purnimaanta* Chaitra) is not quite the same; the solar Chaitra is the *last* (i.e., 12th) month of the year. The lunisolar Chaitra's number is taken by the solar month that begins in it, namely the solar Vaisakha. All this can be seen in the graphic in the next page.

The relationships for all the months may be mapped according to the following table[23]: -

Raasi	Approximate nakshatra on Purnima	Lunar Month Name	Solar Month Name	Assamese Version	Tamil Version	Punjabi Version[24]
Mesha	Chitra	Chaitra	Vaisakha	Bahag	Chittarai	Vaisakh
Vrshava	Visakha	Vaisakha	Jyaistha	Jeth	Vaikasi	Jeth
Mithuna	Jyestha	Jaishta	Aashaadha	Ahar	Aani	Harh
Karkata	(Purva & Uttara) Aashaadha	Aashaadha	Sraavana	Saon	Aadi	Sawan
Simha	Sravana	Sraavana	Bhaadrapada	Bhad	Aavani	Bhadon
Kanya	(Purva & Uttara) Bhaadrapada	Bhaadrapada	Asvayuja (Aasvina)	Ahin	Purattaasi	Asu
Tula	Asvini	Asvayuja (Aasvina)	Kaarthika	Kati	Arppisi	Katik
Vrischika	Krittika	Kaarthika	Maarghasira	Aghon	Karthigai	Maghar
Dhanus	Mrugasira	Maarghasira	Pausa (Pushyam)	Puha	Maargali	Poh
Makara	Pushyami	Pausa (Pushyam)	Maagha	Magh	Thaai	Magh
Kumbha	Maagha	Maagha	Phalguna	Phagun	Maasi	Phagun
Mina	(Uttara and Purva)	Phalguna	Chaitra	Chait	Panguni	Chet

23 Chakravarthy, et al. p. 280
24 Pal Singh Purewal, Nanakshahi Samat. < http://www.sikh.net/sikhism/Nanakshahi.htm >

	Phalguni

The Assamese, Punjabi and Tamil versions have been provided to give an idea of the linguistic variation. It is also interesting to observe that the National Calendar suggested by Saha and Lahiri also uses the same set of month names, increasing the potential confusion. As is probably obvious by now, the rule does not correspond to the Tamil, National and Nanakshahi calendars.

2) Months named after r*aasis*

Only solar months share their names with *raasis*. SK Chatterjee and Apurba Kumar Chakravarthy give the following names along with the associated *raasis*[25].

Raasi	Sanskritised Version	Malayalam Version
Mesha	Mesha	Medam
Vrshava	Vrshava	Edavam
Mithuna	Mithuna	Midhunam
Karkata	Karkata	Karitaka
Simha	Simha	Chingam
Kanya	Kanya	Kanni
Tula	Tula	Thulam
Vrischika	Vrischika	Vrischikam
Dhanus	Dhanus	Dhanu
Makara	Makara	Makaram
Kumbha	Kumbha	Kumbham
Mina	Mina	Minam

That is to say, the month shares its name with that of its corresponding *Sankranti*. For instance, if *Mesha Sankranti* occurs on a certain day, then the period until the next *Sankranti* would be *Mesha maasa* (*Medham maasam).*

This naming rule is followed primarily in the Malayalam calendar. Incidentally, Abhayankar says that the Oriya calendar also follows this rule.[26]

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[25] Chakravarty, et al. p. 280
[26] Abhayankar, p. 55

Parva Dina Nirnaya –
How the days of festivals are decided.

Hearing him speak, she asked,
"The cultural complexity is interesting,
but perhaps you have a festivals listing?
To which Apara Ganita looked at birds chirping and replied thus:-

We provide a list of Indian festivals, along with their (Indic) dates and the calendar used to reckon the particular festival. The list of festivals is by no means exhaustive; the entries are mostly public holidays in India.

Festival[27]	**Indic Date**	**Additional Rules**	**Calendar Used**
Makara Sankranti/ Pongal	Makara Sankranti	None	Solar
Maha Siva Raatri	Magha K 14	Must cover a nisita	Lunisolar
Holi	Phalguna Purnima	*Holika Dahana* is observed on the night of the Purnima; *Holi* is observed on the solar day after *Holika Dahana*	Lunisolar
Ugadi / Gudi Padwa	Chaitra S 1	None	Lunisolar
Rama Navami	Chaitra S 9	Must cover *Madyahna*	
Tamil New Year, Vishu, Bengali New Year	Mesha Sankranti	Respective Sankranti rules	Solar
Ganesh Chaturti	Bhadrapada S 4	Must cover *Madyahna*	Lunisolar
Buddha Purnima	Vaisakhi Purnima		Lunisolar
Raksha Bandan	Sravana Purnima		Lunisolar
Janmashtami	Sravana K 8		Lunisolar

[27] Chatterjee, p. 60-68

Onam	Moon is in Sravana nakshatra in Solar Bhadrapada		Lunisolar and Solar
Mahanavami	Asvayuja S 9	(*Mahanavami* is reckoned before the other 8 days of *Dussehra*[28])	Lunisolar
Vijayadasami	(The thithi after Mahanavami)	Must cover a *Nisita*	Lunisolar
Deepavali	Asvayuja Amavasya	Must cover *pradosha*	Lunisolar

A bit of explanation is necessary. First, the terms. "Nisita" is defined to be a time-period measured by two ghatikas (1/60th of a solar day; approximately 20 minutes) stretching on either side of midnight. "Pradosha" is the time-period stretching for two muhurtas (1/15th of the time between sunrise and sunset; approximately 1 hour 36 minutes) after sunset. "Madhyahna" is one-third of the time-period between sunrise and sunset. This fraction covers mid-day.

Second, these dates are valid only on non-intercalary *thithis* for all lunisolar festivals. Both leap days and non-leap days in leap months are deemed unfit for festivals. (Kshaya maasas are *not* an issue here because a) *jugma* months are deemed fit for religious observance and b) in the Eastern and Northwestern schools, the extra intercalary month is deemed to be normal).

And finally, if the given thithi doesn't cover the given time, or covers the given time on two solar days, then the second solar day is reckoned to be the respective festival.

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[28] Sivasri Sarma, Madugula. Interview by author. Hyderabad, India. 4th January, 2002.

Samvad Sandesha –
How Eras come into play

Perceiving the response, she questioned,
"I don't know if this is an important part,
but from when do all calendars start?"
To which Apara Ganita looked at a foundation stone and replied thus:-

The Indian calendar system follows a wide range of eras, some of historical interest. Also, we do not attempt to link individual calendars to eras, for the same calendar may be reckoned with two different eras in two different places.

Here's the listing[29]: -

Era	Zero Year	Beginning of Era with respect to individual year
Saka	78 CE	Mesha Sankranti, Chaitra S 1
Vikrama	57 CE	Mesha Sankranti, Chaitra S 1, Kartika S 1, Ashadha S 1
Kali	3101 BCE	Mesha Sankranti, Chaitra S 1
Kollam	824 CE	Kanya Sankranti, Simha Sankranti
Bengali San	963 + solar years since 1556 CE	Mesha Sankranti

In addition, some regions also name their years according to the names of the Jovian years. Saha and Lahiri point out that there are two schools for this; the Southern school names its years in continuous succession, while the Northern school names its years corresponding to the present Jovian year[30].

[29] Saha et al. p. 252 – 258.
[30] Ibid. p 272

***Kshaya Sutra* – How certain months are dropped.**

Observing the reaction, she enquired,

To calendars you seem to be an active saakshya[31],
But have you studied the ephemerally confounding kshaya?

To which, Apara Ganita looked at some fallen leaves and replied thus:-

One of the most interesting aspects of the Indian lunisolar calendar is its *kshaya maasas*, literally "decayed months". Occasionally, certain months are dropped from the lunisolar calendar. We now try to understand the modalities behind this omission; we try to answer how, why, when and where it happens.

First, let's try to define a kshaya month. Chatterjee, in his work on Indian calendars, says that a certain lunar month "may completely overlap any of the short three nirayana solar months of Margasira, Pausha and Magha", with the result that there will be no new moon in the respective solar month. Consequently, there will be no lunar month named "after …this solar month".[32] A graphic describing this interaction is given in Appendix C.

We learn the following from this statement: - a) that the solar months of Margasira, Pausa and Magha are small, b) that at a certain time, there might be no new moon in these months, and c) the corresponding lunar month is dropped from the calendar. Note that Chatterjee is silent on whether the dropped lunar month is amaanta or purnimaanta; a naïve assumption would be that since he talks about new moons, the month would be amaanta. But, a study of the (Chaitradi) amaanta and purnimaanta calendars for the present year reveals that the difference between these two calendars is still two weeks. Therefore, it's safe to conclude that kshaya months were dropped from the purnimaanta calendar as well.

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[31] saakshya = witness (in Sanskrit)

[32] Chatterjee, p. 34

Moreover, the statement about "corresponding lunar month" is unclear; are we talking about the lunar month with the same number as the new-moon-lacking solar month? Or are we talking about the lunar month with the same *name* of the solar month? Running the *calendrica* code provided by Dershowitz and Reingold with their book *Calendrical Calculations – The Millenium Edition* (see table for values), we see that it's the lunar month with the same name that gets dropped.

To account for a purnimaanta kshaya, and to further clarify which month to drop, we re-phrase the definition of a kshaya month to be thus: - "in any given lunar year, if two consecutive Sankrantis occur between two consecutive new moons, then the lunar month, whether amaanta or purnimaanta, with the same name as the solar month in which this occurs, is dropped." As we shall see, such a re-phrasing is useful for computational purposes.

Indeed, as we mentioned earlier, we ran the Dershowitz and Reingold's *calendrica* package to get values for the occurrence of a kshaya month. Since searching for a kshaya month is computationally very heavy[33], we used a table prepared by Saha and Lahiri (table 22 in the book)[34] as a starting point. We also tabulated results for non-kshaya months, specifically years with gaps of 19, 46, 65, 76, 122 and 141 years respectively. The results and the graphs from these results are tabulated in the appendix.

It must be noted that all cases tabulated previously have been calculated according to Surya Siddhantic rules and that we may get a different set of results if calculated according to ephemeris calculations. Indeed, as Chatterjee has pointed out, there was a difference in 1964 CE; ephemeris calculations showed Margasira to be kshaya (and

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[33] Dershowitz, Nachum and Reingold. Calendar Tabulations – 1900 to 2200. (2002: Cambridge) Cambridge University Press. p. 24

[34] Saha et al. p. 250

Karthika, Chaitra to be adhika), while as we've seen, Surya Siddhantic computation showed Pausa to be kshaya (and Asvina and Chaitra to be adhika).[35] Chatterjee, however, seems to be in agreement with Dershowitz and Reingold in saying that there was a kshaya in Magha in 1983 CE[36], despite his use of ephemeris calculations.

What do we get from all this? We see that a kshaya month can occur every 19, 46, 65, 76, 122 or 141 years. Indeed, Saha and Lahiri's tabulation provide us with the following frequencies of occurrences for gaps between kshaya months: -

Interval	Number of times occuring
19	11
46	3
65	1
76	1
122	1
141	6

Table – Number of times a particular interval gap occurred

We therefore see that between 525 CE and 1985 CE, kshaya occurred 11 times with a gap of 19 years, thrice with a gap of 46 years, six times with a gap of 141 years, and once each with gaps of 65, 76 and 122 years. The obvious question one would like to ask would be why. Why does kshaya occur only in these gaps?

To answer this better, we re-iterate what causes kshaya in the first place. We already said that a kshaya would occur when two consecutive Sankrantis occur *between* two Amavasyas. That is to say, when a solar month is shorter in length than, and is completely enclosed by, a (an Amaanta) lunar month. Saha and Lahiri go on to say that the "maximum duration of a lunar month exceeds the lengths of the solar months only in

[35] Chatterjee, SK. p. 38

[36] Dershowitz, Nachum and Edward M. Reingold. Calendrical Calculations – The Millennium Edition. (2001: Cambridge) Cambridge University Press. p. 269

the case of Margasira, Pausa and Magha"[37] and that, therefore, kshaya is possible only in these months.

This would explain the solar month part, but what of lunar? How can the lunar month be bigger than the solar month? Ala'a Juwad has some answers; in his article, he suggests that the canonical synodic month, a lunar month between two consecutive phases of the moon, is not constant in length. Indeed, he goes on to say that between 1600 and 2400 CE, the synodic month extends in length from 29 days 6 hours and 31 minutes to 29 days 19 hours and 59 minutes.[38] Moreover, he says that the "longest lunar months … occur when the date of the new Moon coincides with apogee".[39] A brute-force search for the longest synodic month definitely won't give us a kshaya; for kshaya to occur, the lunar month needs to be only bigger than its solar counterpart and more importantly, completely encompass it. Indeed, Jawad says that the longest synodic month occurred in 1610 CE, a year which occurs within the 141 year long kshaya hiatus between 1540-1541 CE and 1680 – 81 CE.

We therefore search for other clues to unscramble kshaya. On a purely arithmetic perspective, we observe the following: -

19	= 19 * 1
46	= 19 * 2 + 8
65	= 19 * 3 + 8
76	= 19 * 4
122	= 19 * 6 + 8
141	= 19 * 7 + 8

That is to say, the year-gaps are in the form 0, 8 mod 19.

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[37] Saha et al. p. 250

[38] Jawad, Ala'a. "How Long Is a Lunar Month?" in Sky & Telescope, November 1993. p. 76

[39] Ibid.

Is it possible then, that the kshaya month has something to do with the Metonic cycle? The Metonic Cycle is a fairly well documented phenomenon; first observed by the Greek astronomer Metos, every 19 years, the lunar dates overlap with the tropical ones. The underlying mathematical reason is simple: - 19 sidereal years contain 19*365.242189 = 6939.6 solar days, while 235 synodic months (with a mean of 29.53 solar days) contain 235*29.530588853 = 6939.68 solar days. The lengths overlap. But this obviously is neither necessary nor sufficient; it might be useful for the dates to repeat, but it definitely doesn't fulfil the requirement for kshaya.

One suggestion therefore, might be that the kshaya occurs when the number of solar days of a sidereal year is equal to that of a synodic month, which in turn is equal to that from an anomalistic month. An anomalistic month is defined to be the time – period between two consecutive perigee passages and has a mean value of 27.55455 days. Taking these average values, we calculate the average values of solar days in whole numbers of synodic and anomalistic months (canonical kshaya years shaded for reference): -

Interval	Occurrence	Modulo	Solar Year	Synodic Months	Anomalistic Months
19	11	1*19	6939.601591	6939.68838	6943.7466
27	0	1*19+8	9861.539103	9863.216677	9864.5289
38	0	2*19	13879.20318	13879.37676	13887.4932
46	3	2*19+8	16801.14069	16802.90506	16808.2755
57	0	3*19	20818.80477	20819.06514	20831.2398
65	1	3*19+8	23740.74229	23742.59344	23752.0221
76	1	4*19	27758.40636	27758.75352	27774.9864
84	0	4*19+8	30680.34388	30682.28182	30695.7687
95	0	5*19	34698.00796	34698.4419	34718.733
103	0	5*19+8	37619.94547	37621.9702	37639.5153
114	0	6*19	41637.60955	41638.13028	41634.92505
122	1	6*19+8	44559.54706	44561.65858	44555.70735
133	0	7*19	48577.21114	48577.81866	48578.67165
141	6	7*19+8	51499.14865	51501.34696	51499.45395

Broadly speaking, we might summarize the above table as thus: - for the most part, the number of solar days in solar years, synodic and anomalistic months overlap in

kshaya years. However, this overlap doesn't occur *only* in kshaya years; as the table shows, there's an overlap for 133 years as well. Does this, then, *explain* the kshaya phenomenon? We might summarize it as being strongly suggestive, but definitely not conclusive.

Treatment of Kshaya Months[40]

We may complete our discussion of kshaya months by describing the three Kshaya Schools of thought.

The *North Western School* is followed in the north-western part of the country, presumably in Gujarat and/ or Rajasthan, where the lunisolar calendar is used. Essentially, the North Western School treats the adhika month before kshaya as a normal month and the one after the kshaya month to be intercalary. This contrasts with the *Eastern School* where the reverse is followed; the adhika month before the kshaya is deemed intercalary, while the one after it is deemed normal. The Eastern School is followed in the eastern parts of the country, where the lunisolar calendar is followed. The final of the trio, the *Southern School,* treats both adhika maasas as intercalary, instead reckoning the kshaya month as a "jugma", i.e., the first half of the thithi of this month is deemed to be that of the first month, and the second half as that of the second month. This is presumably followed in the Southern parts of the country where the lunisolar calendar is followed.

[40] Chatterjee, SK. p 37- 40

Epilogue – The Beginning

By this time, onlookers all sides gathered around the two. They were attentively listening to the conversation between them. Along with Apara Ganita, they were waiting for the dwara palika to ask once again. But she didn't. She stood and smiled. Her face was radiant, glowing like the moon on a Purnima and the harsh summer sun entering the Mithuna raasi.

She still said nothing. She got up and walked away from the crowd. Still smiling. Still graceful.

The sparks came slowly, but suddenly. All around them, the landscape was changing. The gate was melting into the walls, the walls into the ground. The ground was changing into grass, the grass covering the entire ground.

Except the ground underneath Apara Ganita. He found himself standing on an elevated podium, facing listeners all around him, all waiting to hear him speak.

For once, he didn't know what to say.

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Acknowledgements

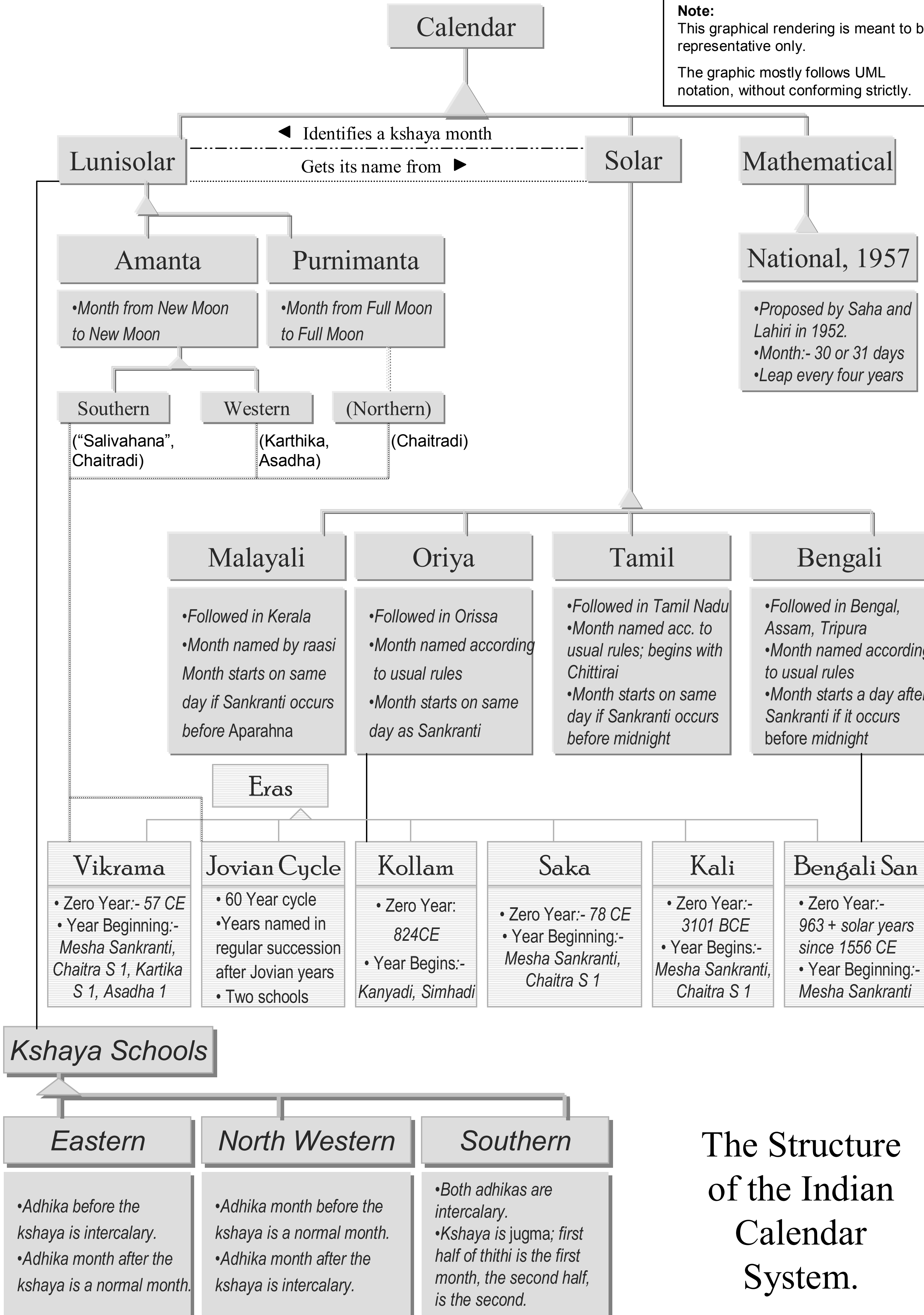
This report is 5150 words long, making it the biggest report I've ever written. It's been in the making for the last one-year in two countries. Obviously, there are many people who've helped me, and I'd like to thank everyone of them.

First, I'd like to thank Dr. Helmer Aslaksen, my research supervisor; sir, it's been a pleasure working with you. It was great synergy all the way. Akhil Deogar and Akshay Prasad, the other two students who worked on this, also need a grateful thank-you here. Way to go guys, we made it. To Dr. Deshowitz, for helping me out with the *calendrica* code, just when I was stuck. To Dr. Subramanyam and all the wonderful people at the Department of Astrology, Potti Sriramulu Telugu University, Hyderabad, it's been a pleasure meeting you all and I'm grateful you sat through my presentation on that cold December evening. To Dr. BG Siddharth, Director General, BM Birla Science Center, your comments were invaluable. To Dr. Vallabh, Professor and Head, Astronomy Department, Osmania University, sir, I thank you for spending some time with me. To Dr. Madugula Sivasri Sarma, it's been a pleasure meeting you and I'm grateful for your comments on the rules for festivals. To Dr. CVL Narasimham, for those wonderful evenings on the banks of the River Musi discussing panchangam traditions. And last but not least, I'd like to thank my parents and my little brother for keeping up with me during all those late-night sessions I spent pouring over calendars.

I once again thank everyone who's helped me. Of course, it bears no need to say that all errors are mine.

Note:
This graphical rendering is meant to be representative only.

The graphic mostly follows UML notation, without conforming strictly.

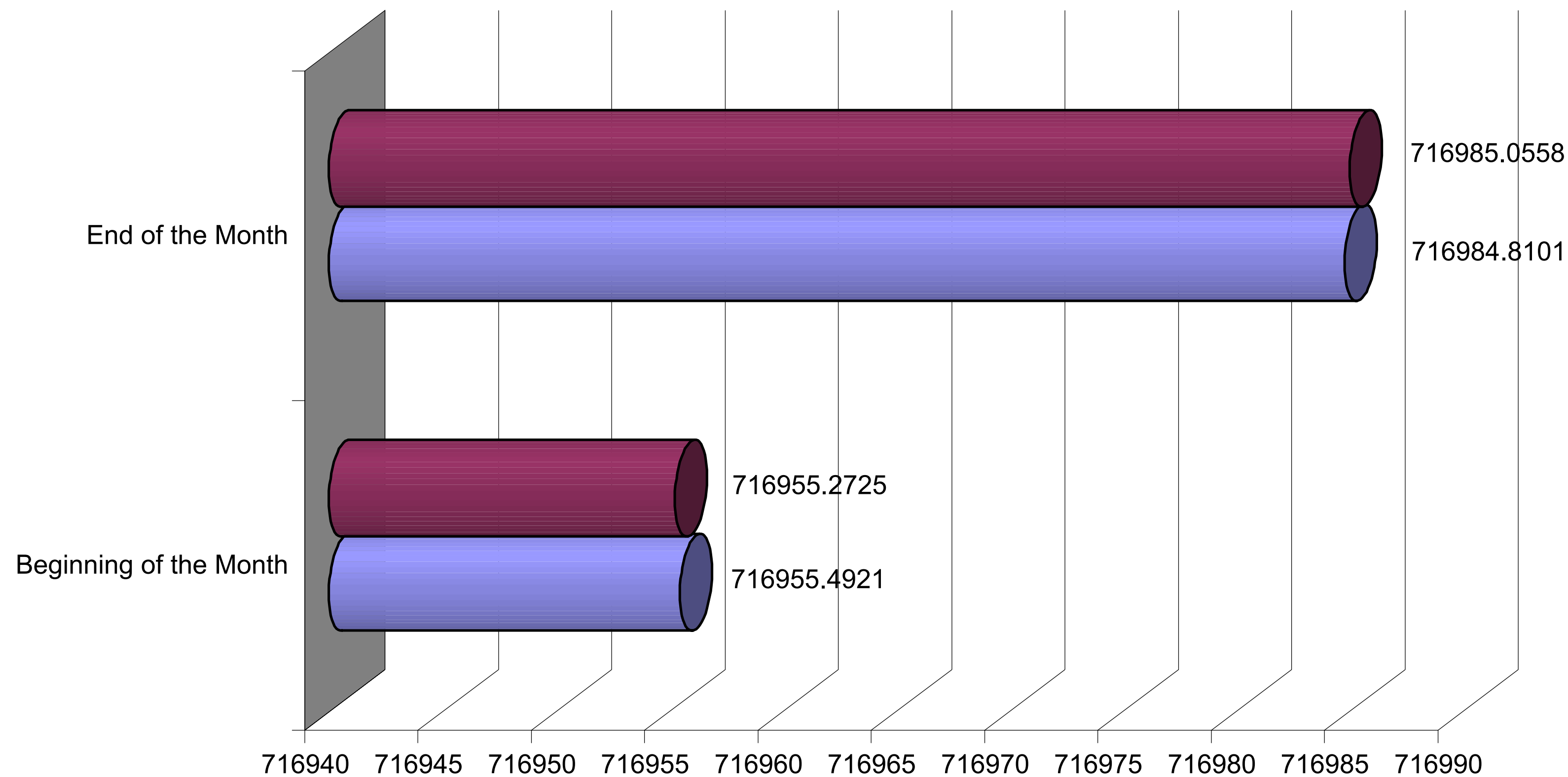


The Structure of the Indian Calendar System.

Hindu Solar (Saka Era)	Fixed Day	Gregorian (CE)	Hindu Lunar (Vikrama Era)	Leap Month?	Leap Day?	Sankranti (hr:min:sec)	Sankranti (Moments)	Amavasya (hr:min:sec)	Amavasya (Moments)
1885/9/1	716955	1963/12/16	2020/09/01	FALSE	FALSE	11:48:39.726	716955.4921	06:32:25.892	716955.2725
1885/10/1	716984	1964/01/14	2020/09/30	FALSE	FALSE	19:26:34.354	716984.8101	01:20:25.252	716985.0558
1744/9/1	665453	1822/12/13	1879/8/30	FALSE	FALSE	00:10:27.526	665454.0073	17:52:00.161	665453.7444
1744/10/1	665483	1823/1/12	1879/9/30	FALSE	FALSE	07:48:32.677	665483.3254	13:46:57.632	665483.5743
1603/9/1	613952	1681/12/10	1738/9/1	FALSE	FALSE	12:32:25.884	613952.5225	05:14:33.480	613952.2184
1603/10/1	613981	1682/1/8	1738/9/30	FALSE	FALSE	20:10:20.531	613981.8405	01:11:37.994	613982.0497
1462/9/1	562450	1540/12/8	1597/8/30	FALSE	FALSE	00:54:13.716	562451.0377	17:36:31.986	562450.7337
1462/10/1	562480	1541/1/7	1597/9/30	FALSE	FALSE	08:32:08.341	562480.3557	12:29:37.136	562480.5206
1443/8/1	555481	1521/11/9	1578/8/30	TRUE	FALSE	15:07:49.331	555481.6304	14:22:17.756	555481.5988
1443/9/1	555510	1521/12/8	1578/8/29	FALSE	TRUE	02:54:38.328	555511.1213	09:59:51.334	555511.4166
1397/10/1	538738	1476/1/6	1532/9/30	FALSE	FALSE	12:52:28.218	538738.5364	11:39:20.696	538738.4857
1397/11/1	538767	1476/2/4	1532/10/30	FALSE	FALSE	23:36:42.517	538767.9838	00:05:00.597	538768.0035
1321/9/1	510949	1399/12/6	1456/8/30	FALSE	FALSE	13:16:01.519	510949.5528	07:32:11.950	510949.314
1321/10/1	510978	1400/1/4	1456/9/30	FALSE	FALSE	20:53:56.115	510978.8708	00:39:59.510	510979.0278
1302/8/1	503979	1380/11/5	1437/8/30	TRUE	FALSE	03:29:37.170	503980.1456	02:34:46.447	503980.1075
1302/9/1	504009	1380/12/5	1437/8/30	FALSE	FALSE	15:16:26.115	504009.6364	21:15:12.204	504009.8856
1256/9/1	487207	1334/12/5	1391/8/30	FALSE	FALSE	17:36:21.429	487207.7336	11:17:01.381	487207.4702
1256/10/1	487236	1335/1/3	1391/9/30	FALSE	FALSE	01:14:16.038	487237.0516	03:48:04.528	487237.1584
1237/8/1	480238	1315/11/6	1372/8/1	FALSE	FALSE	07:50:07.603	480238.3265	05:47:04.872	480238.241
1237/9/1	480267	1315/12/5	1372/8/30	FALSE	FALSE	19:36:46.043	480267.8172	19:48:11.479	480267.8251
1904/10/01	723924	1983/01/14	2039/09/30	FALSE	FALSE	17:26:09.739	723924.7265	09:03:57.421	723924.3777
1904/11/01	723953	1983/02/12	2039/10/30	FALSE	FALSE	04:10:13.481	723954.1738	04:45:33.318	723954.1983

How the Amavasya Escaped Pausa in 1964 CE.

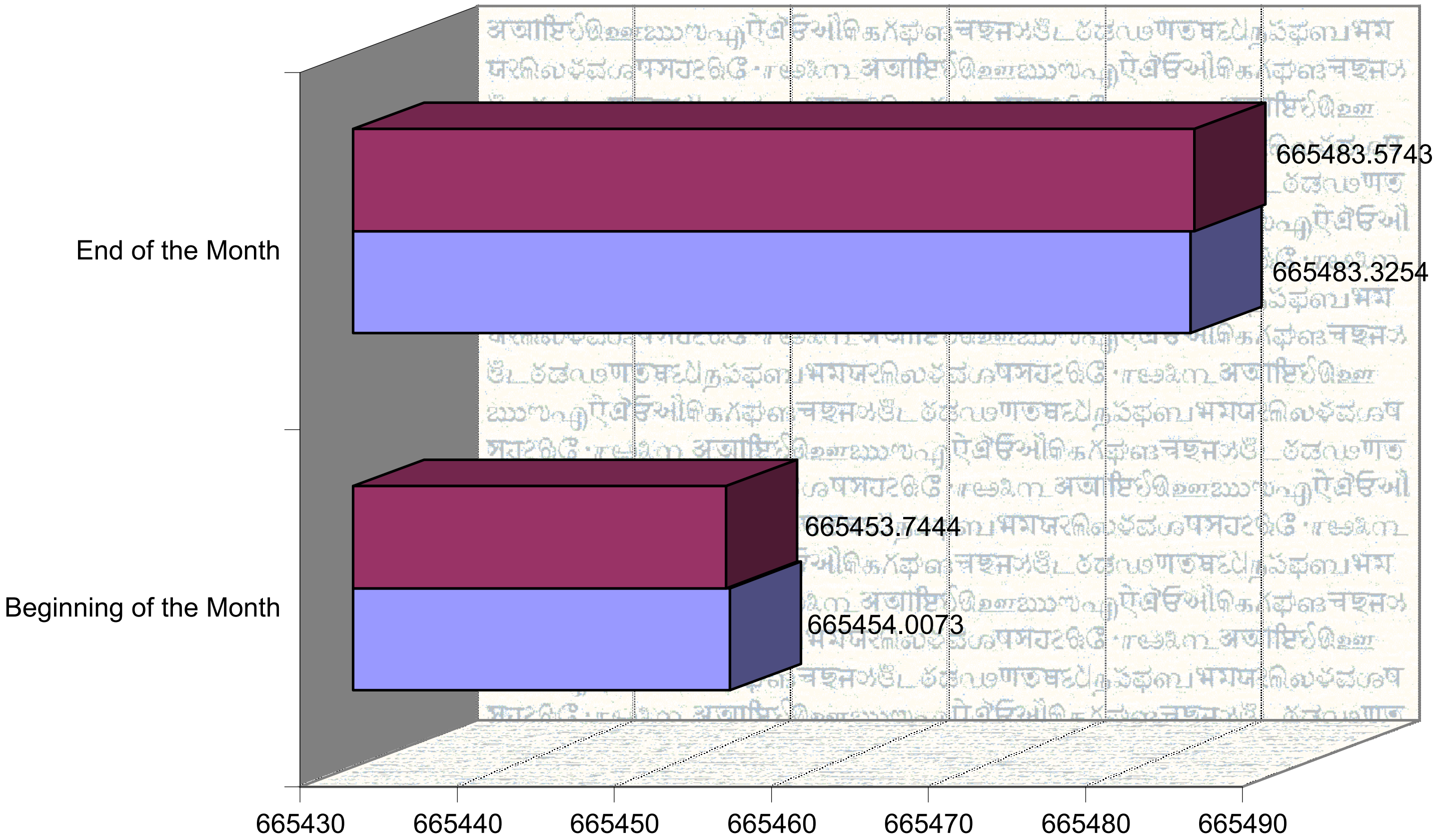
All times expressed in Rational Moments.



	Beginning of the Month	End of the Month
Amavasya	716955.2725	716985.0558
Samkranti	716955.4921	716984.8101

How the Pausa decayed in 1823 CE.

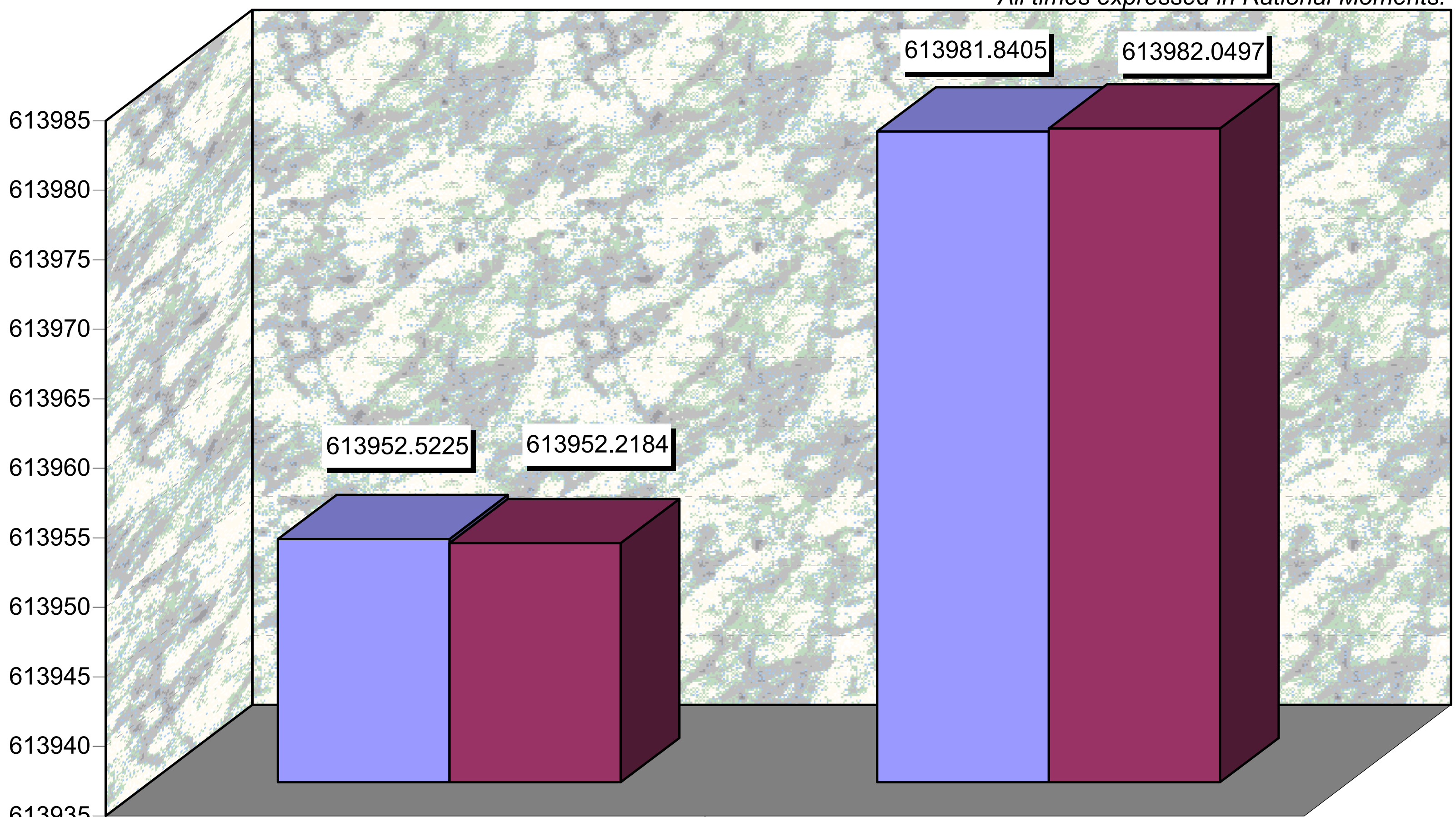
All times expressed in Rational Moments.



	Beginning of the Month	End of the Month
Amavasya	665453.7444	665483.5743
Samkranti	665454.0073	665483.3254

How Pausa Disappeared in 1682 CE.

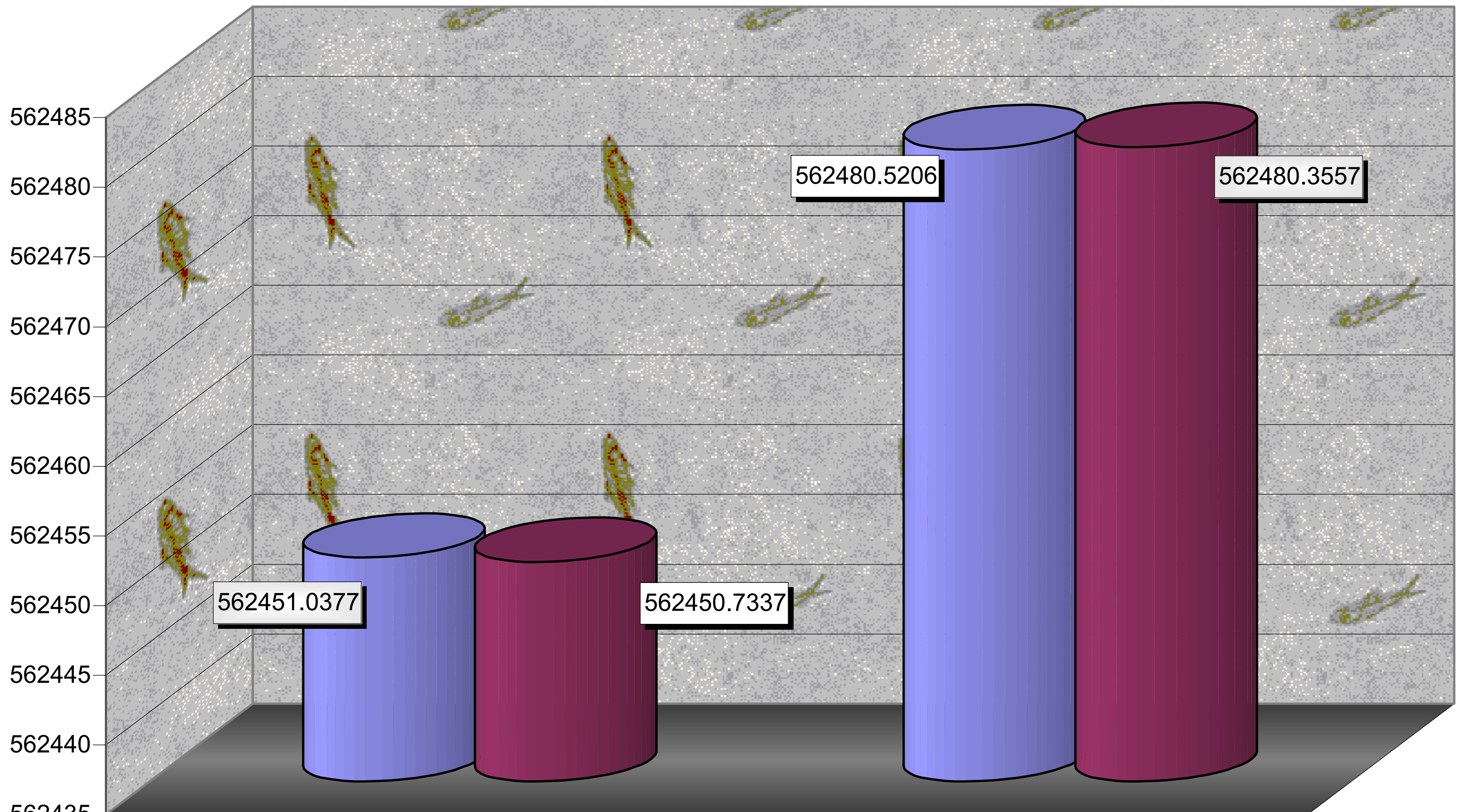
All times expressed in Rational Moments.



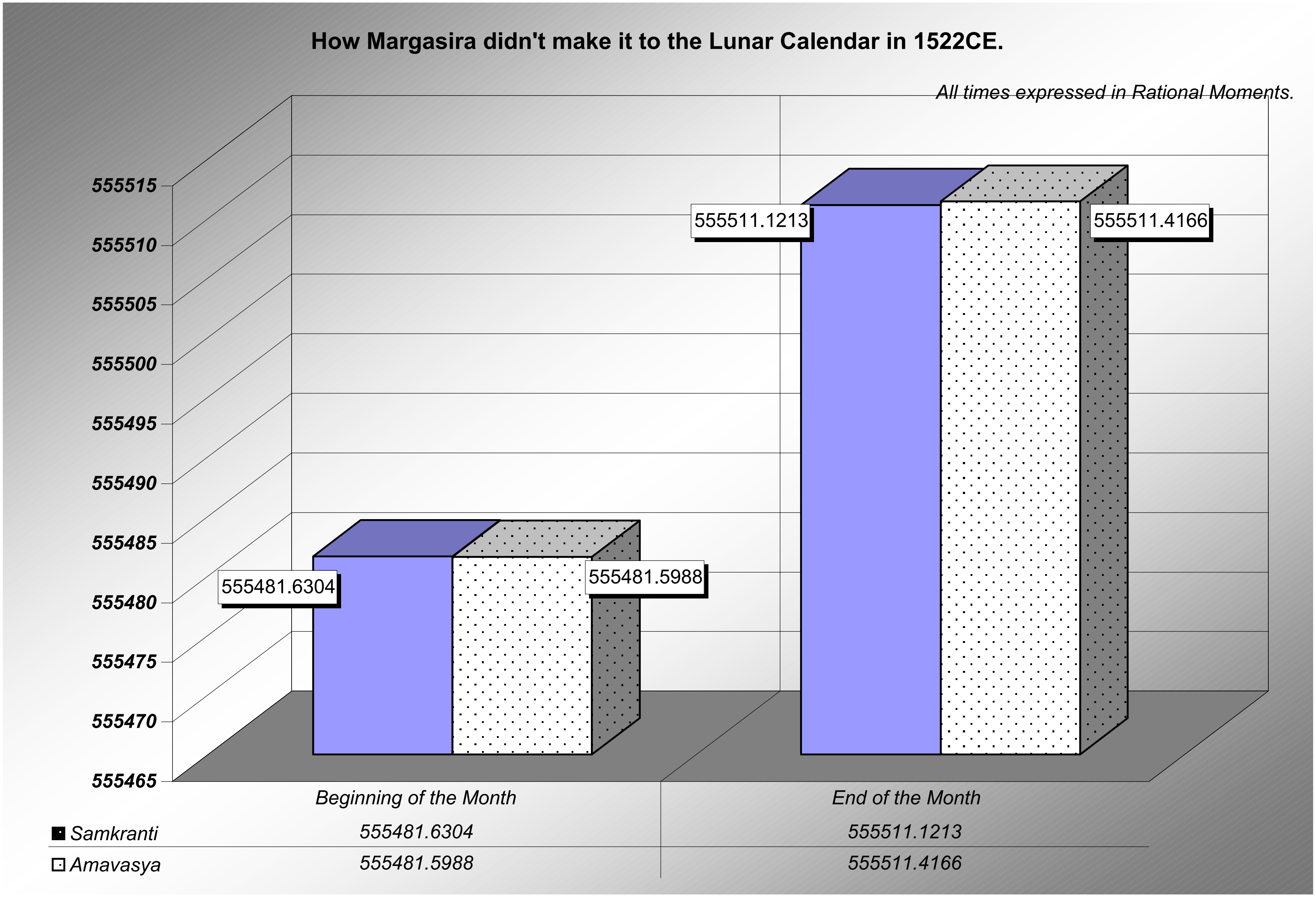
	Beginning of the Month	End of the Month
Samkranti	613952.5225	613981.8405
Amavasya	613952.2184	613982.0497

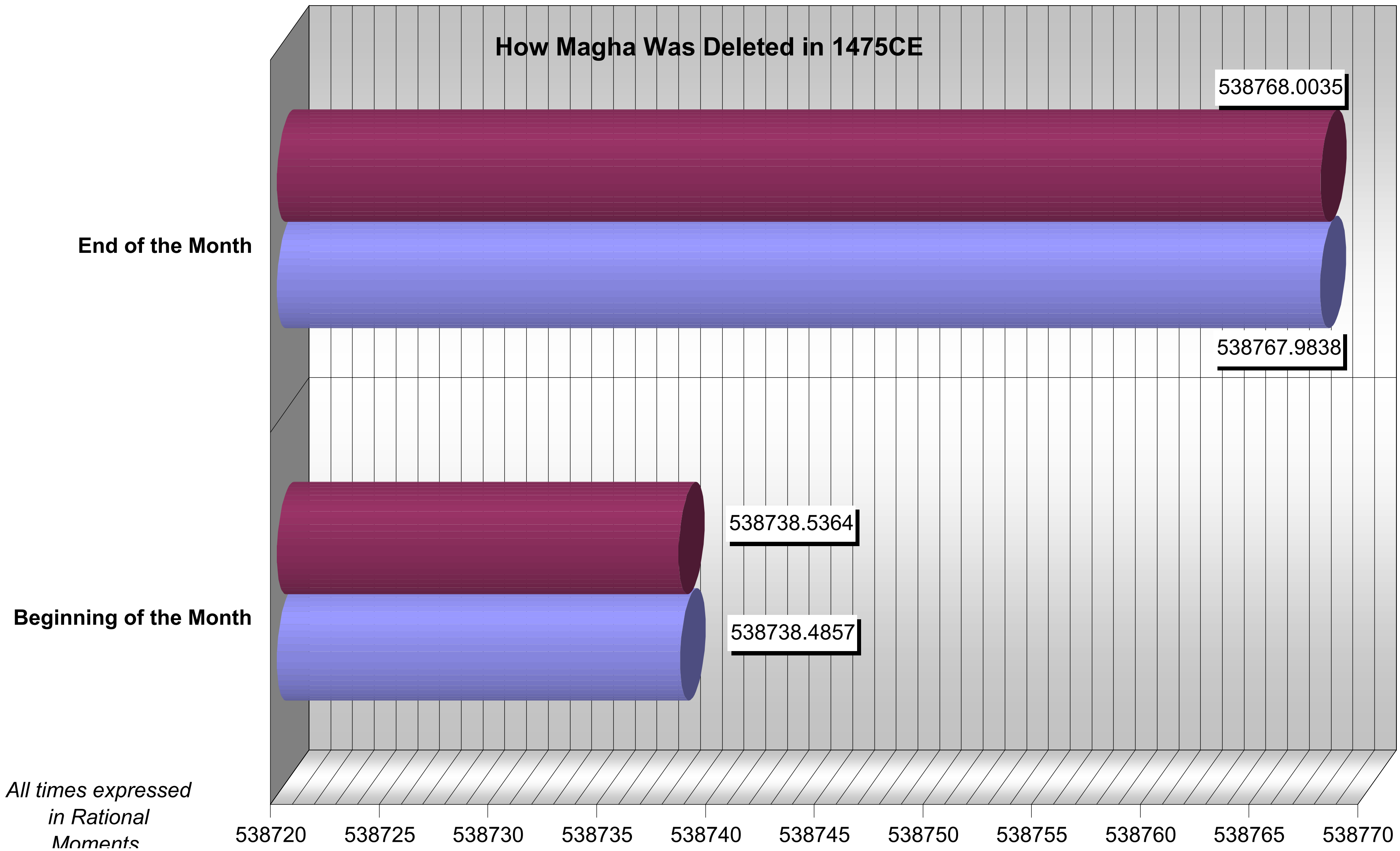
How Pausa Was Dropped in 1541 CE

All times expressed in Rational Moments.



	Beginning of the Month	End of the Month
Samkranti	562451.0377	562480.3557
Amavasya	562450.7337	562480.5206

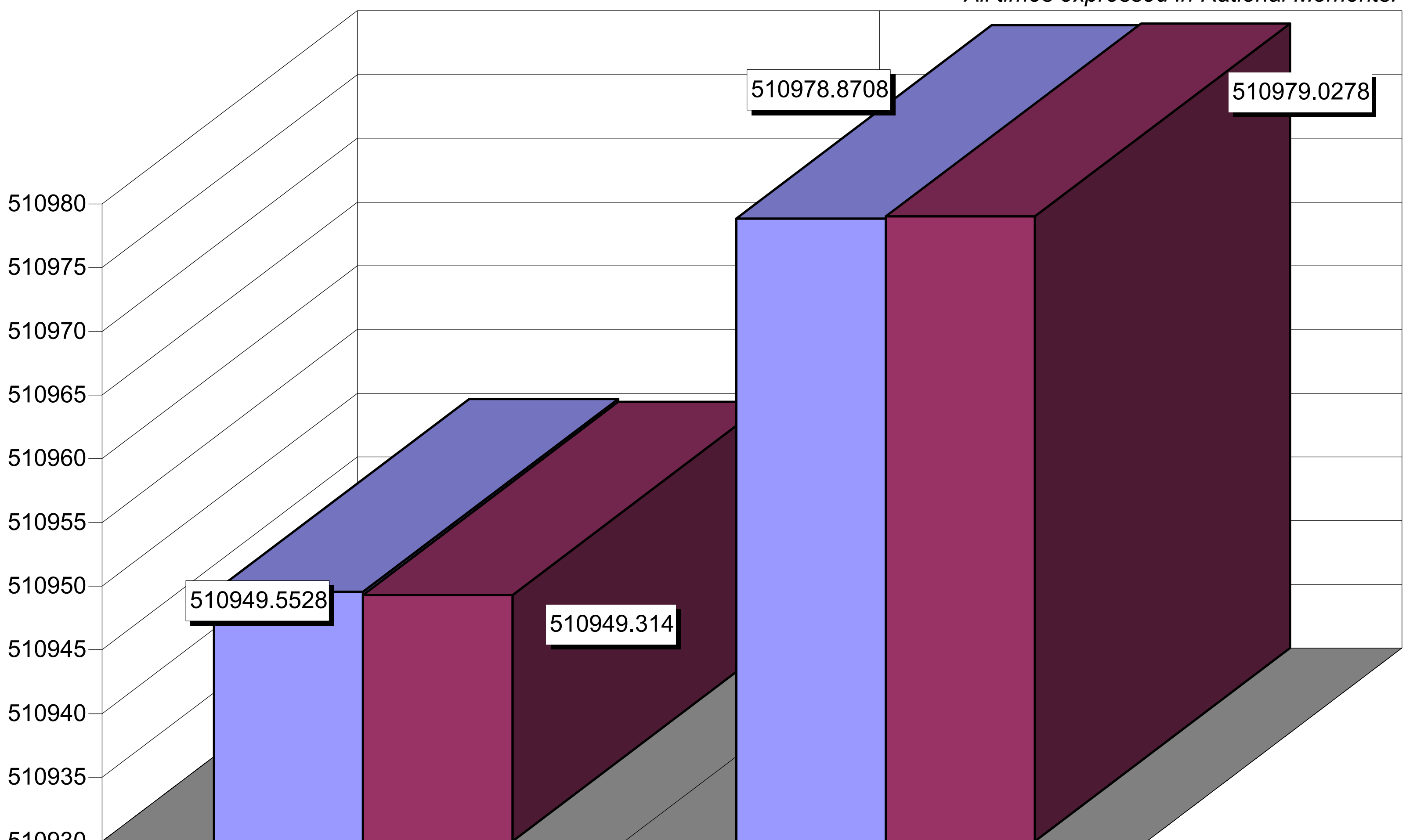

How Margasira didn't make it to the Lunar Calendar in 1522CE.
All times expressed in Rational Moments.
555515
555510
555505
555500
555495
555490
555485
555480
555475
555470
555465
555481.6304
555481.5988
555511.1213
555511.4166
Beginning of the Month
End of the Month
Samkranti
555481.6304
555511.1213
Amavasya
555481.5988
555511.4166



	Beginning of the Month	End of the Month
Amavasya	538738.4857	538768.0035
Samkranti	538738.5364	538767.9838

How Pausa Ran Away in 1399-1400CE

All times expressed in Rational Moments.



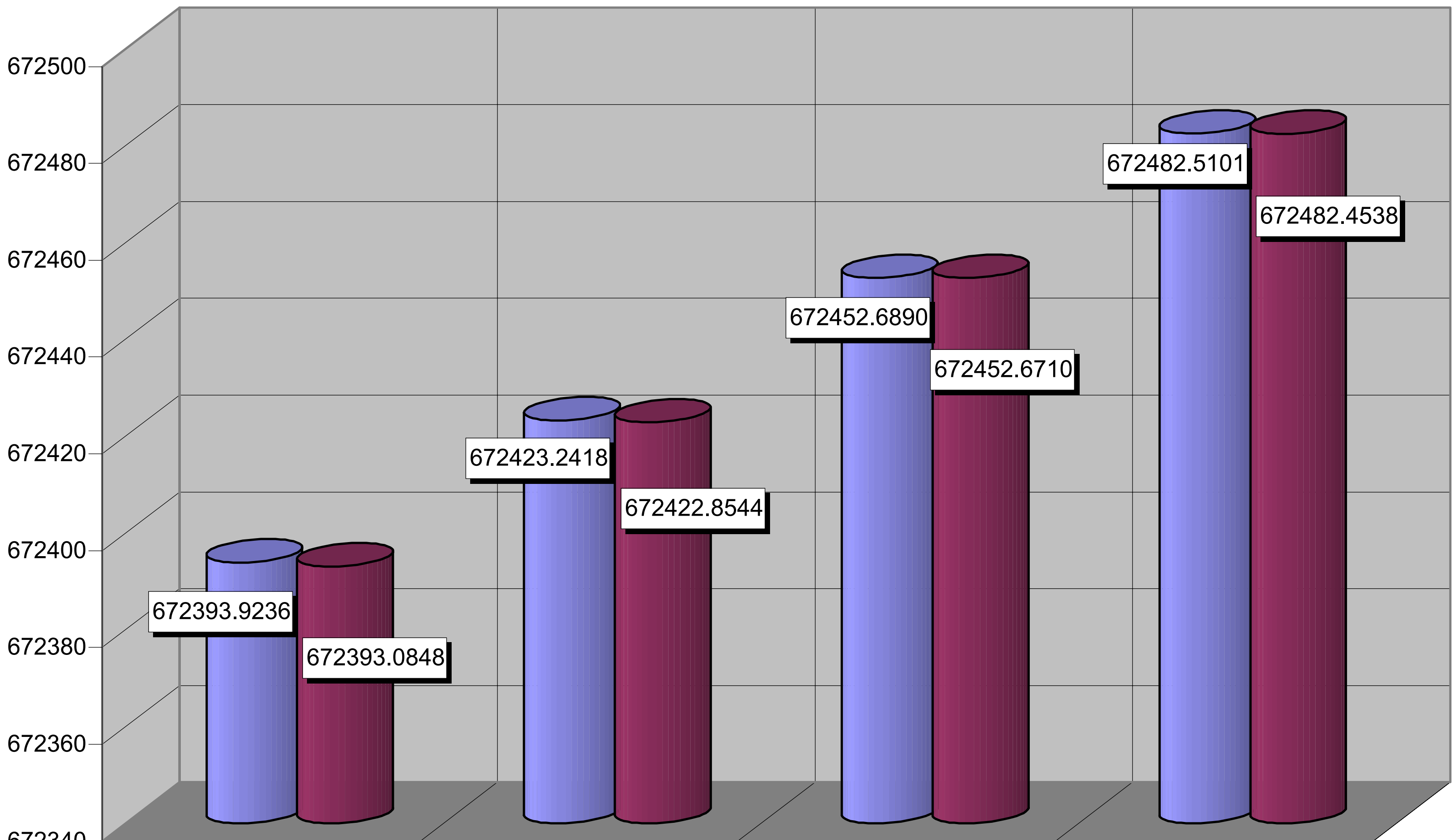
	Beginning of the Month	End of the Month
Samkranti	510949.5528	510978.8708
Amavasya	510949.314	510979.0278

Hindu Solar	Fixed Date	Gregorian	Hindu Lunar	Leap Month?	LeapDay?	Sankranti		Amavasya	
1744/9/1	665453	1822/12/13	1879/8/30	FALSE	FALSE	00:10:27.526	665454.0073	17:52:00.161	665453.7444
1744/10/1	665483	1823/1/12	1879/9/30	FALSE	FALSE	07:48:32.677	665483.3254	13:46:57.632	665483.5743
1763/9/1	672393	1841/12/13	1898/9/1	FALSE	FALSE	22:10:02.920	672393.9236	02:02:04.954	672393.0848
1763/10/1	672422	1842/1/11	1898/9/30	FALSE	FALSE	05:48:08.110	672423.2418	20:30:22.804	672422.8544
1763/11/1	672452	1842/2/10	1898/10/30	FALSE	FALSE	16:32:11.852	672452.6890	16:06:10.978	672452.6710
1763/12/1	672482	1842/3/12	1898/11/30	FALSE	FALSE	12:14:30.094	672482.5101	10:53:27.932	672482.4538
1790/9/1	682255	1868/12/13	1925/09/30	FALSE	FALSE	21:50:32.239	682255.9101	06:09:24.227	682256.2565
1790/10/1	682284	1869/1/11	1925/10/29	FALSE	FALSE	05:28:37.402	682285.2282	23:17:43.494	682285.9706
1790/11/1	682314	1869/2/10	1925/11/29	FALSE	FALSE	16:12:41.144	682314.6755	18:05:11.108	682315.7536
1790/12/1	682344	1869/3/12	1925/12/29	FALSE	FALSE	11:54:48.881	682344.4964	13:00:01.825	682345.5417
1809/9/1	689195	1887/12/14	1944/09/30	FALSE	FALSE	19:50:18.190	689195.8266	00:36:07.304	689196.0251
1809/10/1	689224	1888/1/12	1944/10/29	FALSE	FALSE	03:28:12.797	689225.1446	13:35:00.591	689225.5660
1809/11/1	689254	1888/2/11	1944/11/30	FALSE	FALSE	14:12:16.539	689254.5919	04:16:12.013	689255.1779
1809/12/1	689284	1888/3/12	1944/12/30	FALSE	FALSE	09:54:34.781	689284.4129	20:22:28.399	689284.8489
1820/9/1	693213	1898/12/14	1955/09/01	FALSE	FALSE	16:08:59.667	693213.6729	17:13:40.842	693212.7178
1820/10/1	693242	1899/1/12	1955/10/01	FALSE	FALSE	23:46:54.268	693242.9909	04:05:28.667	693242.1705
1820/11/1	693272	1899/2/11	1955/11/01	FALSE	FALSE	10:31:08.567	693272.4383	14:31:57.701	693271.6055
1820/12/1	693301	1899/3/12	1955/12/01	FALSE	FALSE	06:13:16.303	693302.2592	00:51:14.516	693301.0356
1866/9/1	710015	1944/12/15	2001/09/30	FALSE	FALSE	13:49:04.331	710015.5757	19:41:30.672	710015.8205
1866/10/1	710044	1945/01/13	2001/10/29	FALSE	FALSE	21:27:09.517	710044.8939	10:16:11.944	710045.4279
1866/11/1	710074	1945/02/12	2001/11/30	FALSE	FALSE	08:11:13.259	710074.3411	22:40:48.473	710074.9450
1866/12/1	710103	1945/03/13	2001/12/29	FALSE	FALSE	03:53:20.996	710104.1620	09:06:03.927	710104.3792
1885/9/1	716955	1963/12/16	2020/09/01	FALSE	FALSE	11:48:39.726	716955.4921	06:32:25.892	716955.2725
1885/10/1	716984	1964/01/14	2020/09/30	FALSE	FALSE	19:26:34.354	716984.8101	01:20:25.252	716985.0558
1885/11/1	717013	1964/02/12	2020/11/29	FALSE	FALSE	06:10:38.096	717014.2574	17:47:57.629	717014.7416
1885/12/1	717043	1964/03/13	2020/12/29	FALSE	FALSE	01:52:56.390	717044.0784	07:20:04.127	717044.3056

Between 1841 and 1964 CE:-

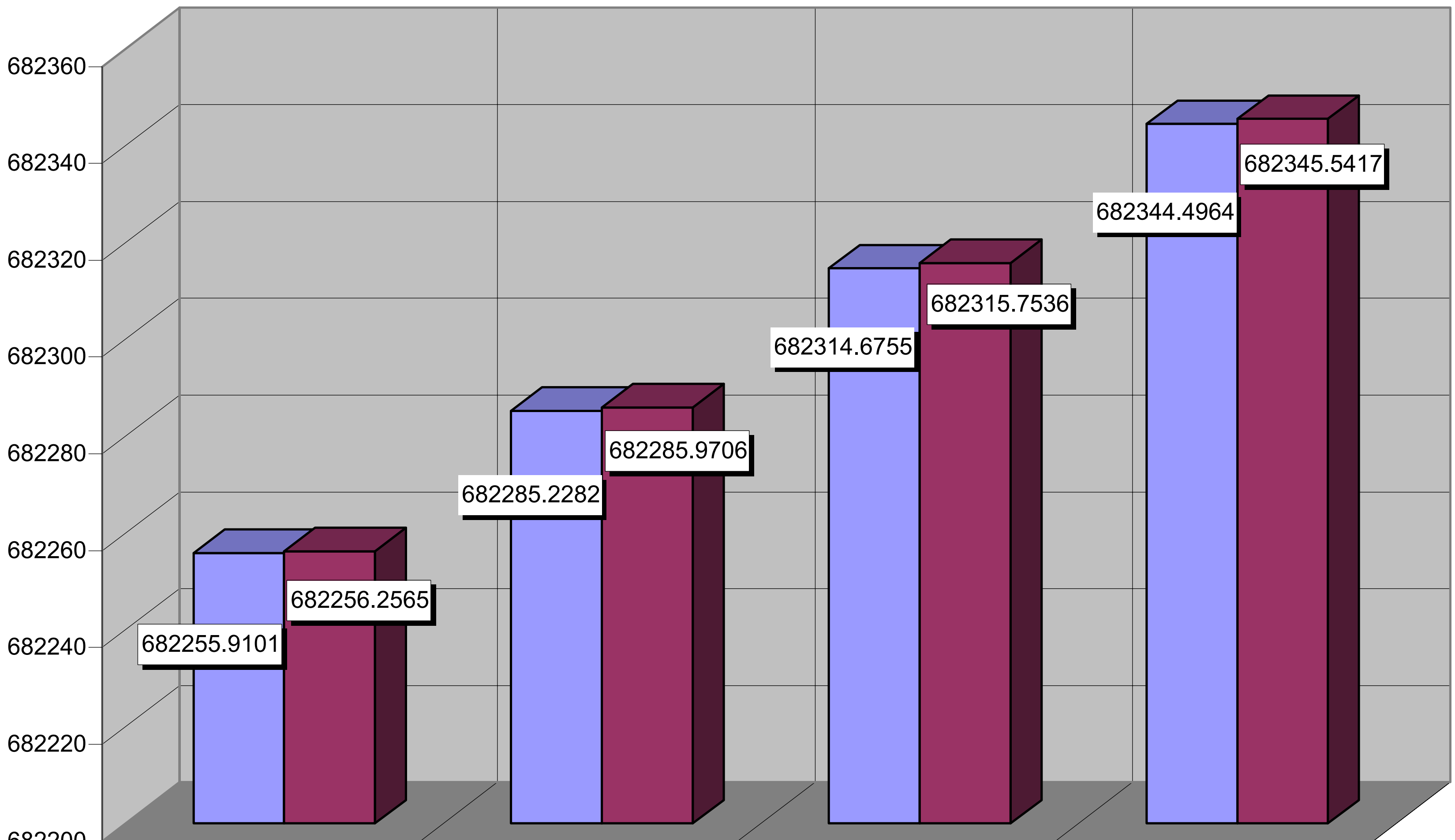
A Tabulation of Samkranti and Amavasya Timings for Margasira, Pausa, Magha and Phalguna, 19, 46, 63, 76, 122 and 141 years after 1763 Saka Era.

How there was no kshaya in 1841 CE.



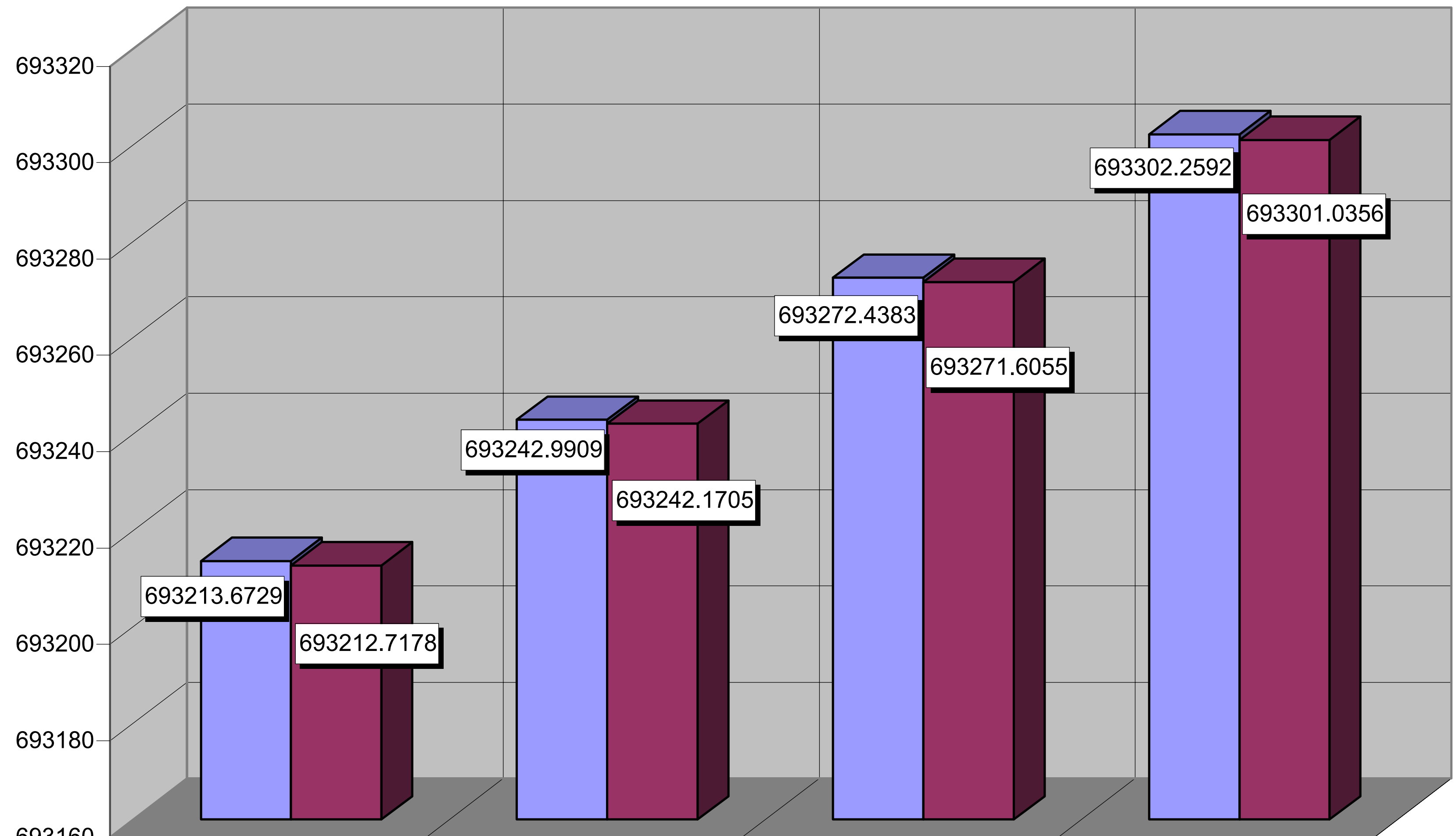
	Margasira	Pushya	Magha	Phalguna
Sankranti	672393.9236	672423.2418	672452.6890	672482.5101
Amavasya	672393.0848	672422.8544	672452.6710	672482.4538

How Kshaya Never Took Place in 1868-69 CE.



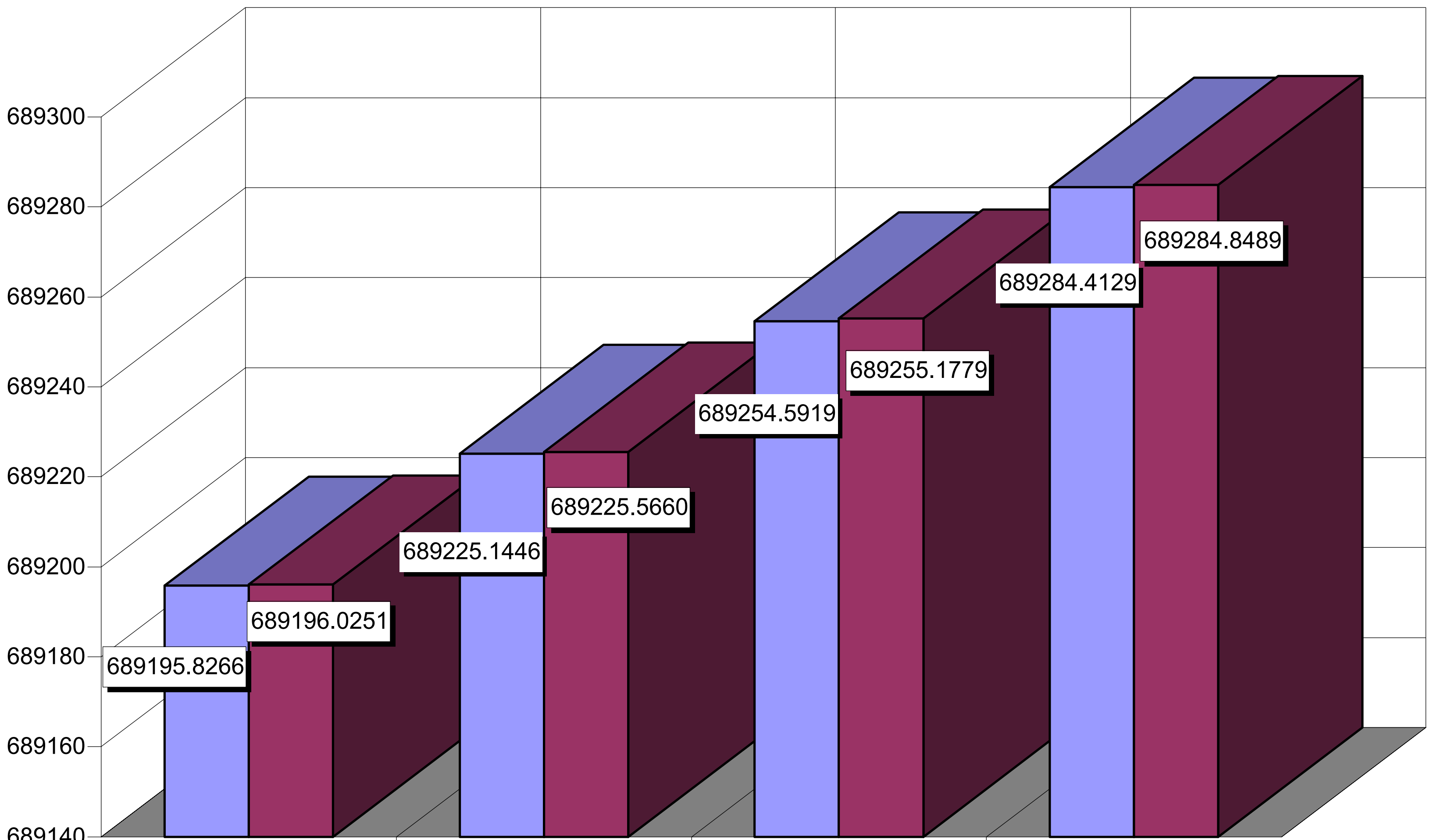
	Margasira	Pushya	Magha	Phalguna
Sankranti	682255.9101	682285.2282	682314.6755	682344.4964
Amavasya	682256.2565	682285.9706	682315.7536	682345.5417

How all months made it to the calendar in 1898-99 CE.



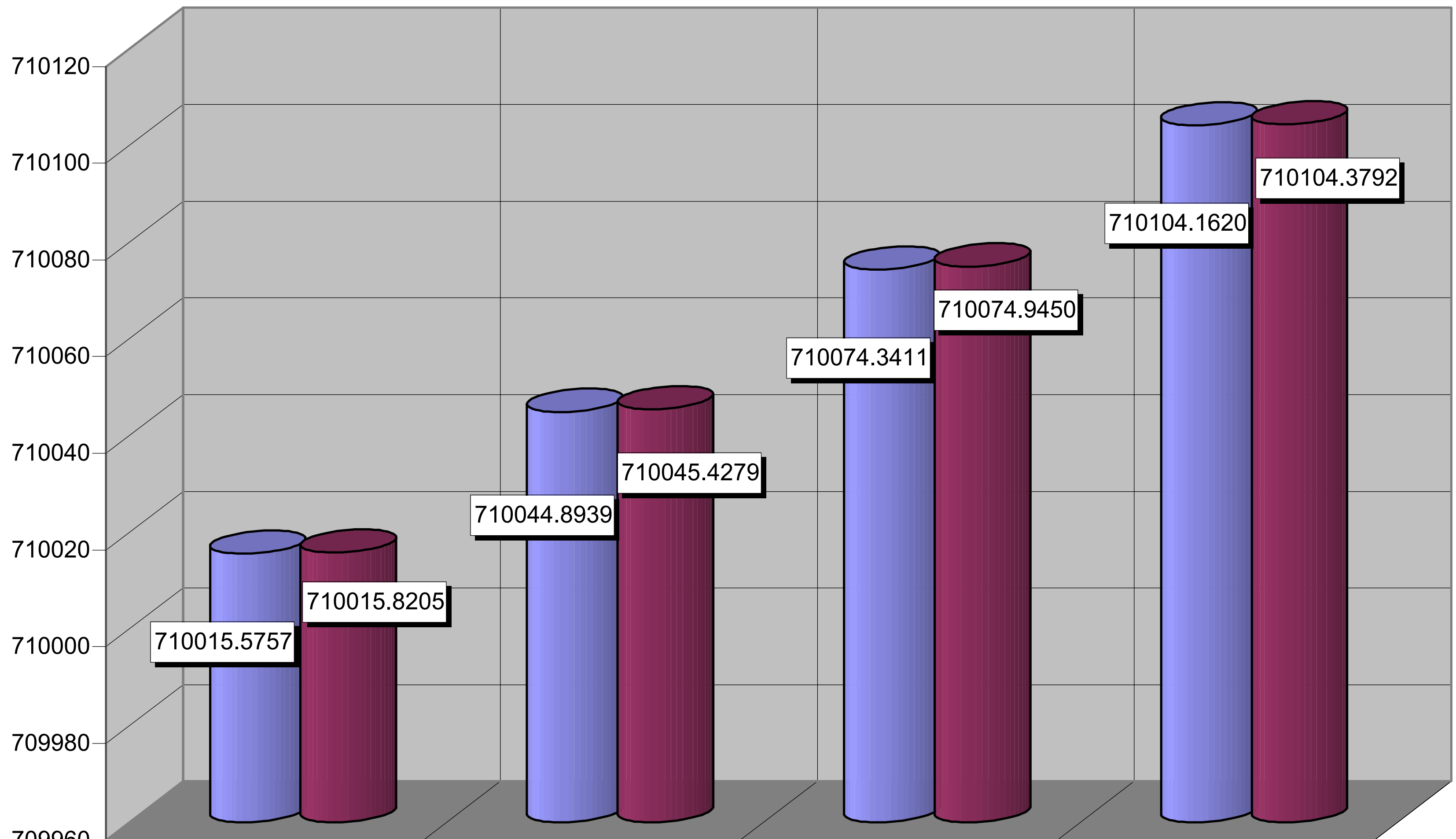
	Margasira	Pushya	Magha	Phalguna
Sankranti	693213.6729	693242.9909	693272.4383	693302.2592
Amavasya	693212.7178	693242.1705	693271.6055	693301.0356

How No Month Got Decayed in 1887-88 CE



	Margasira	Pushya	Magha	Phalguna
Sankranti	689195.8266	689225.1446	689254.5919	689284.4129
Amavasya	689196.0251	689225.5660	689255.1779	689284.8489

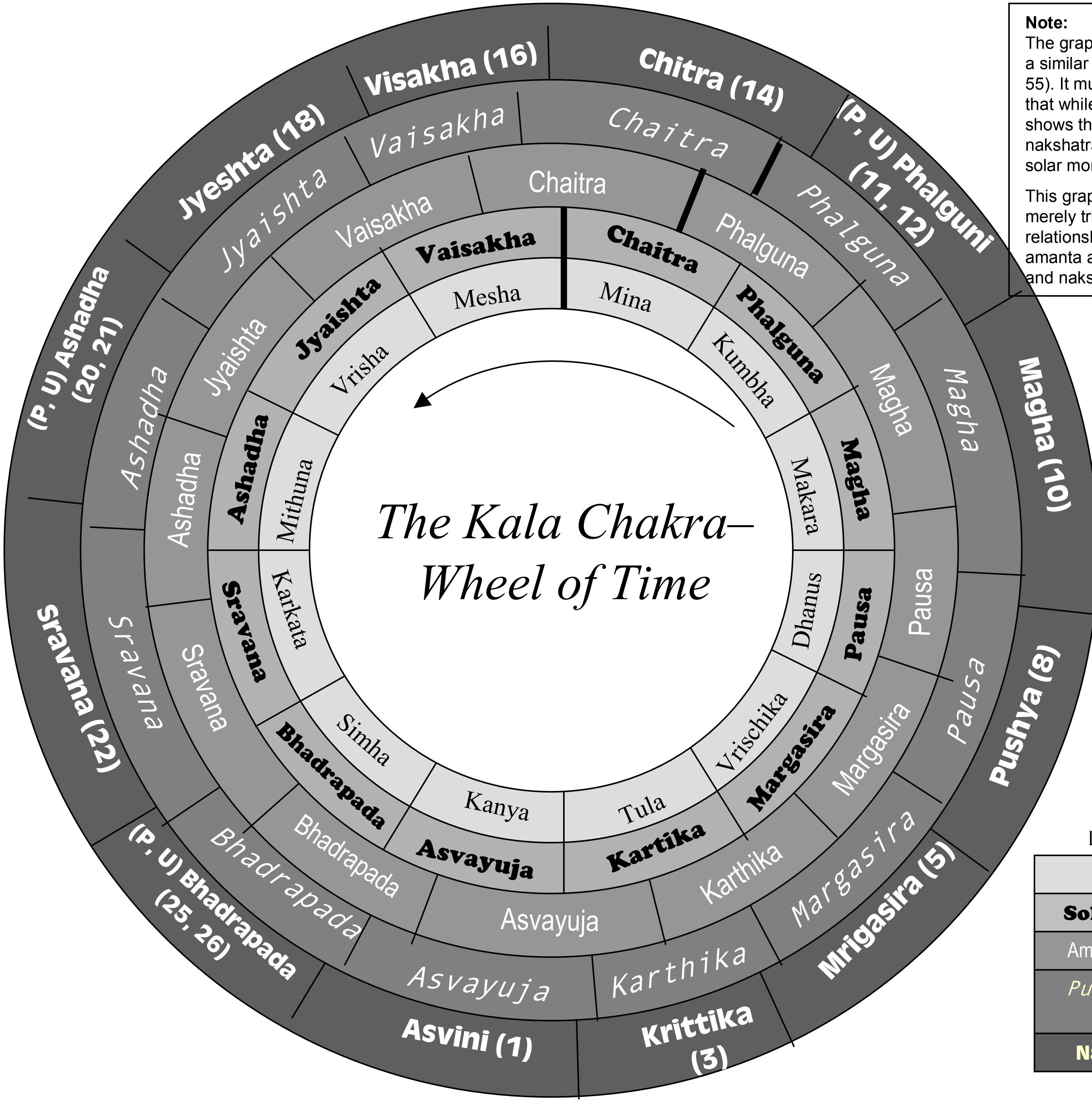
How there were no subtractions to the calendar in 1944-45 CE.

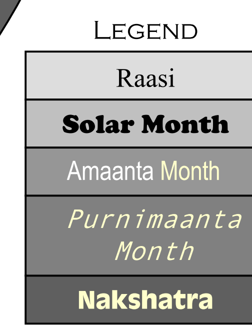


	Margasira	Pushya	Magha	Phalguna
Samkranti	710015.5757	710044.8939	710074.3411	710104.1620
Amavasya	710015.8205	710045.4279	710074.9450	710104.3792

Note:
The graphic is mostly inspired by a similar diagram in Chatterjee (p. 55). It must be, however, noted that while the Chatterjee diagram shows the "relative dispositions of nakshatra divisions, raasis and solar months", by plotting the sky.

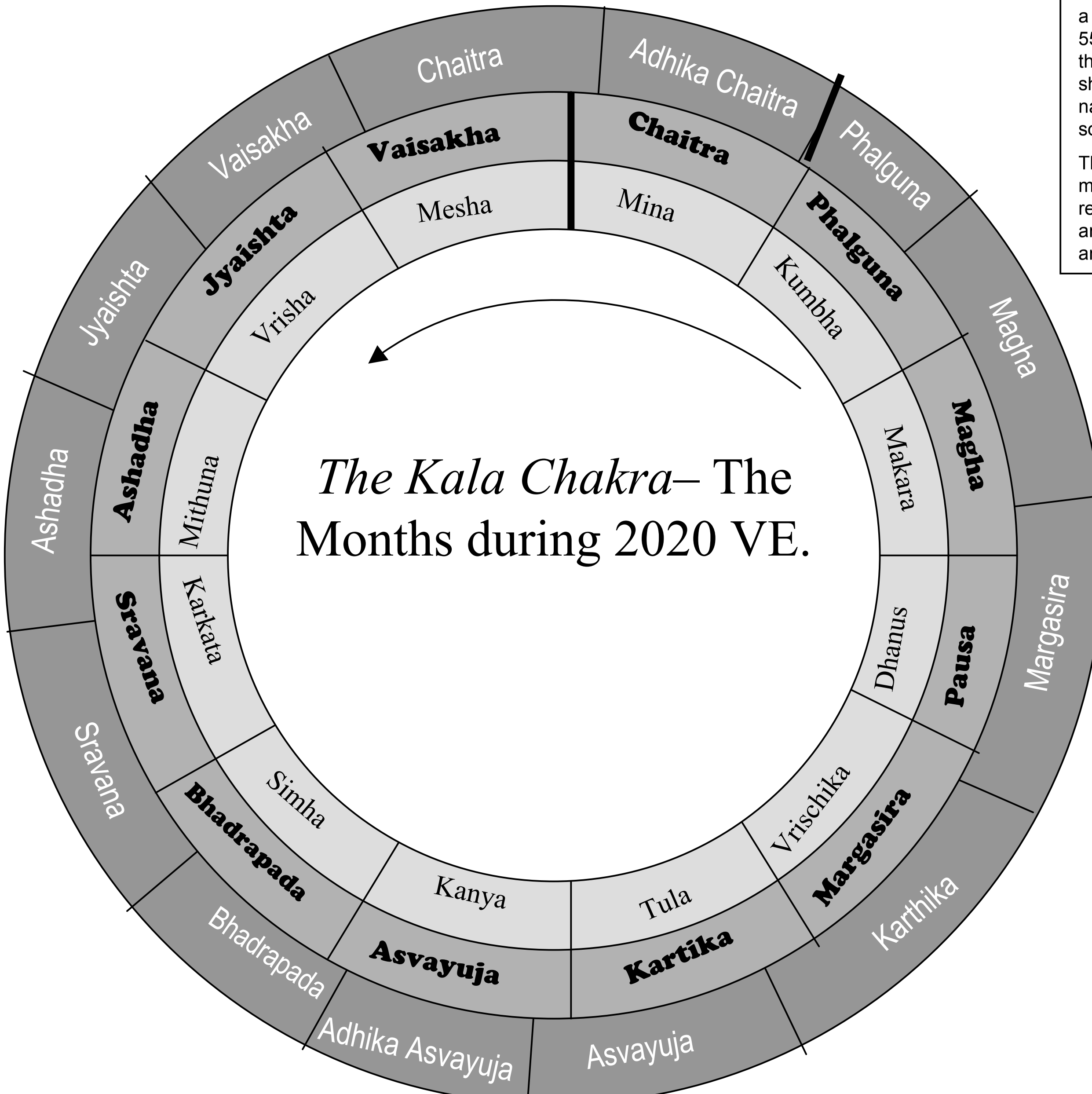
This graphic, on the other hand, merely tries to show the relationship between raasis, solar, amanta and purnimanta months and nakshtras.





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The graphic is mostly inspired by a similar diagram in Chatterjee (p. 55). It must be, however, noted that while the Chatterjee diagram shows the “relative dispositions of nakshatra divisions, raasis and solar months”, by plotting the sky.

This graphic, on the other hand, merely tries to show the relationship between raasis, solar, amanta and purnimanta months and nakshtras.



Legend

Raasi
Solar Month
Amanta Month